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Orientifold points in M theory

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Abstract: We identify the lift to M theory of the four types of orientifold points, and show that they involve a chiral fermion on an orbifold fixed circle. From this lift, we compute the number of normalizable ground states for the SO(N) and Sp(N) supersymmetric quantum mechanics with sixteen supercharges. The results agree with known results obtained by the mass deformation method. The mass of the orientifold is identified with the Casimir energy.

Keywords: M-Theory, D-branes, M(atrix) Theories.

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1. Introduction

In this note we shall be considering the supersymmetric matrix quantum mechanics with 16 supercharges. The lagrangean is

$$\mathcal{L} = \frac{1}{2g^2} tr \left[\dot{X}^i \dot{X}^i + 2\theta^T \dot{\theta} - \frac{1}{2}[X^i, X^j]^2 - 2\theta^T \gamma_i [\theta, X^i] \right], \tag{1.1}$$

where both X^i, $1 \leq i \leq 9$, and the fermionic variables θ, a nine dimensional spinor, are in the adjoint representation of some Lie algebra G.

The case $G = \mathrm{SU}(N)$ is the most well studied [1, 2, 3]. This is the lagrangean describing N D0 branes in type IIA string theory (we shall ignore the decoupled U(1) piece). The matrix theory conjecture [4] asserts that this lagrangean, in the large N limit, describes M theory in the light cone frame.

Since M theory contains a single massless superparticle, there should be a normalizable threshold bound state for each value of N (corresponding to the momentum N graviton). Hence M theory and especially matrix theory predict that there should be precisely one ground state for $G = \mathrm{SU}(N)$, i.e. the Witten index should be 1. This prediction was tested and confirmed by a detailed calculation of the Witten index [5, 6].

The calculation of the Witten index for an arbitrary Lie group was performed by Kac and Smilga [7]. Their analysis, to be reviewed in section 2, uses the mass deformation method, due to Witten [8], and Porrati and Rozenberg [9]. We consider their results for the cases $G = \mathrm{Sp}(N), \mathrm{SO}(N)$ where they can be written in terms of a partition function

$$\sum_{N=1}^{\infty} d_N q^N = \prod_{r}^{\infty} (1 + q^r)\,, \tag{1.2}$$

where r runs over even/odd positive integers for $\mathrm{Sp}(N/2)/\,\mathrm{SO}(N)$ respectively, and d_N is the supersymmetric index. Our goal is to provide an M theory explanation for the results of [7], or alternatively, an independent derivation of their result without making the assumption on which the method of mass deformation hinges.

In section 3 we provide a realization of this lagrangean with $G = \mathrm{Sp}(N)$, $\mathrm{SO}(N)$ as a system of N D0 branes of type IIA string theory moving in the background of one of the four types of orientifold points, $\mathrm{O0}^+$, $\mathrm{O0}^-$, $\widetilde{\mathrm{O0}}$, and $\widehat{\mathrm{O0}}$. We claim that these backgrounds lift to M theory orbifolds on $\mathbb{R}^9/\mathbb{Z}_2 \times \mathbf{S}^1$. We argue that the results of [7] can be phenomenologically reproduced by postulating a chiral fermion living on the fixed circle (the projection of the fixed circle onto $\mathbb{R}^9/\mathbb{Z}_2$ is the fixed point at the origin). Periodic (R) boundary conditions on the circle correspond to the Sp groups while anti-periodic (NS) boundary conditions correspond to SO groups.

Independent evidence for this proposal was given by the results of Dasgupta and Mukhi [10] , who argued, by an analysis of gravitational anomalies of M theory on $\mathbf{T}^9/\mathbb{Z}_2$ (with (0,16) supersymmetry in 1+1 dimensions), that the orbifold fixed line carries a chiral fermion in a supersymmetry singlet, in beautiful agreement with our phenomenology.

It remains to explain how the different boundary conditions on periodicity R/NS correlate with the group family Sp / SO. Following a discussion in [11], we compare the known masses of the $\mathrm{O0}^\pm$ orientifolds with the R/NS Casimir energy in M theory and find complete agreement. We find that $\mathrm{O0}^+$ and $\widehat{\mathrm{O0}}$, which both give an $\mathrm{Sp}(N)$ group, are related through the ground state degeneracy in the Ramond sector. Thus in addition to rederiving the counting of bound states, we find a physical picture for the various orientifold points. It would be nice to get an M theory picture also for the discrete torsions of [12].

We conclude with remarks on the connection to Matrix theory in section 5. Matrix theory compactification on $\mathbf{T}^9/\mathbb{Z}_2$ was also considered in [13], where results consistent with ours were obtained.

2. A review of the mass deformation method

In this section we will review the results and arguments of [7]. We can think of the quantum mechanics (1.1) as a dimensional reduction from 4 dimensions. Using four dimensional $\mathcal{N} = 1$ language, the matter content is a vector multiplet with 3 chiral multiplets, $\Phi_i,\ 1 \leq i \leq 3$ in the adjoint representation, and the superpotential is

$$W = g\epsilon^{ijk} f_{ABC} \Phi_i^A \Phi_j^B \Phi_k^C\,, \tag{2.1}$$

where ϵ is the antisymmetric tensor and f are the structure constants of G. Following [9] the superpotential is deformed by a mass term

$$\Delta W = M \Phi_i^A \Phi_i^A \,. \tag{2.2}$$

The idea is that it is relatively easy to find zero energy solutions to the deformed lagrangean. One needs to solve for the D and F equations, thereby reducing the problem to algebra, and then count the number of solutions. These vacua break the gauge group completely. For large M all degrees of freedom are very massive and the quantum wavefunction is concentrated around the classical solution, and so is normalizable. As we take the limit $M \to 0$ these wavefunctions deform continuously, move towards the center at $\Phi = 0$ and become wider. If one assumes (without proof) that all these states remain normalizable in the limit, then we have a count of the number of bound states which is easier than a computation of the Witten index. For SU(N) one gets the correct result, and our independent derivation for Sp / SO suggests further that this assumption is valid.

The F and D equations are

$$\begin{aligned} \epsilon_{ijk} f^{ABC} \Phi_i^A \Phi_j^B &\propto \frac{M}{g} \Phi_k^C \,, \\ f^{ABC} \Phi_i^A \bar{\Phi}_i^B &= 0 \,. \end{aligned} \tag{2.3}$$

To these we add the condition that the vevs break the gauge group completely, namely that the normalizer in G of the subgroup spanned by the Φ_i is trivial. These equations can be interpreted [14] to describe embeddings of sl(2) in the (complexified) group G which have a trivial centralizer.

For the unitary groups there is a unique solution to these equations, as expected. The solution can be identified with the N dimensional representation of SU(2) (or sl(2)). In general the question can be translated in mathematical terms to a classification of "distinguished markings" for a Lie algebra. The results are

a) For SO(N) the index is the number of partitions of N into distinct odd parts.

b) For Sp(N) the index is the number of partitions of $2N$ into distinct even parts (Sp(1) = SU(2)).

The solution corresponding to a given partition can be identified with the reducible representation of SU(2) with the given partition into irreducible components.

Anticipating ourselves, we note that the number of bound states, d_N can be conveniently coded in a partition function $Z = \sum_{N=1}^{\infty} q^N d_N$

$$\begin{aligned} Z_{\mathrm{SO}(N)} &= \prod_{r=1}^{\infty} (1 + q^{2r-1}) \,, \\ Z_{\mathrm{Sp}(N)} &= \prod_{r=1}^{\infty} (1 + q^{r}) \,. \end{aligned} \tag{2.4}$$

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We realize that phenomenologically, these partition functions exactly represent a chiral fermion on a circle. For SO groups we have half integer moding (NS), while for Sp it is integral (R).

3. Physical realization

In order to get a physical realization of the results of [7] we should find a physical system for the lagrangean (1.1) with $G = \mathrm{Sp}(N), \mathrm{SO}(N)$. As $\mathrm{Sp}(N) \subset \mathrm{SU}(2N)$, $\mathrm{SO}(N) \subset \mathrm{SU}(N)$ we start with a system of N D0 branes that carries an $\mathrm{SU}(N)$ group, and use an orientifold point to project onto the required group.

3.1 Orientifolds

Let us review briefly the properties of orientifold planes in string theory. An orientifold plane (Op plane) is a background for string theory where we mod out by reversing the coordinates transverse to the Op plane and reverse the string orientation at the same time. This background has 16 supersymmetries. To specify the action on open strings one needs to fix the action on the Chan-Paton indices, and there are two ways to do that. The two kinds of O planes are called O^+ and O^-, and the corresponding open string gauge groups are Sp and SO. An $\mathrm{O}p^\pm$ plane carries RR charge and tension according to $Q = T = \pm 2^{p-5}$. This is determined by looking at a string diagram on a test Dp brane.

Actually, one can distinguish a total of 4 kinds of orientifolds as follows. From here on we shall specialize to O0's. (A similar discussion on this point for the case of orientifold 3-planes may be found in [12].) The O0 point is surrounded by an $\mathbb{RP}^8$. We can introduce two $\mathbb{Z}_2$ valued Wilson lines, denoted b and c, corresponding to nontrivial gauge configurations of the B_{NS} two-form and a C_{RR} 5-form (the dual of the 3-form). The four types of orientifold planes differ in the values of (b, c). $b = 0$ gives SO groups and $b = 1$ gives Sp groups, from the ordinary coupling of B_{NS} to the worldsheet.

Let us enumerate the different O0 points and their properties. Our normalization for the RR charge is that a D0 with an image has charge $+1$, while a D0 stuck at the orientifold has charge $+1/2$.

The $\mathrm{O0}^-$ has a trivial value in both $\mathbb{Z}_2$'s, $(b, c) = (0, 0)$, gives an $\mathrm{SO}(2N)$ gauge group and carries a RR gauge field charge $-1/32$.

The $\mathrm{O0}^+$ has $(b, c) = (1, 0)$ and the $\widehat{\mathrm{O0}}$ has $(1, 1)$. Both carry RR charge $+1/32$ and give $\mathrm{Sp}(N)$ groups.

The $\widetilde{\mathrm{O0}}$ has $(b, c) = (0, 1)$, has a D0 stuck on it and thus carries a RR charge $+15/32$ and gives $\mathrm{SO}(2N + 1)$ groups.

In summary, branes which realize an $\mathrm{Sp}(N)$ gauge theory have RR charge $N + 1/32$ and branes which realize an $\mathrm{SO}(N)$ gauge theory have RR charge $N/2 - 1/32$.

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3.2 A phenomenological fermion

The lagrangean (1.1) with SO(N) and Sp(N) gauge groups can therefore be realized by N D0 branes at an orientifold point. To analyze this system, we need its M theory interpretation.

When we go from type IIA to M theory, we have an extra circle. The orientifold $\mathbb{R}^9/\mathbb{Z}_2$ in IIA will then lift to a space $\mathbb{R}^9/\mathbb{Z}_2 \times \mathbf{S}^1$. We will assume that the $\mathbb{Z}_2$ does not act on the $\mathbf{S}^1$ factor (this assumption will be justified in the next section, and also by the consistency of our analysis). Hence there is a singular $\mathbf{S}^1$ at the origin of the $\mathbb{R}^9$ coordinates.

Suppose there was a field living on the singular orientifold circle. This field can then be decomposed into momentum modes along this circle. A mode with mode number N along the circle is interpreted in type IIA as a bound state of N D0 branes sitting at the orientifold point. We therefore have a bound state for each allowed mode.

Now suppose that there are a total of N D0 branes in the system. We can partition this number N into any number of pieces $N = n_1 + n_2 + \cdots + n_m$. We can then form a state which is a m-particle state, the first particle being a bound state of n_1 D0 branes, the second particle being a bound state of n_2 D0 branes, and so on. These states are localized in $\mathbb{R}^9$ and hence are normalizable. Hence it contributes to the Witten index. The number of ground states is then the number of partitions of N.[1]

However, [7] found that we actually need the partitions to be distinct integers. This has a natural interpretation: the state at the orientifold point should be fermionic.

Furthermore, these states only exist if we have D0 branes bound to the orientifold point, and not anti-D0 branes, because a state with an orientifold point and anti-D0 branes is not supersymmetric. This implies that the fermion can have only positive mode number i.e. it must be a chiral fermionic field.

What about the multiplet structure? A standard D0 brane is a BPS state and breaks 16 of the 32 supersymmetries of type IIA. These broken supersymmetries acting on the D0 brane generate a $2^8 = 256$ dimensional multiplet of states: the graviton multiplet in 11 dimensions.

Here however, we have a D0 brane along with an orientifold point. The orientifold point preserves the same supersymmetries as a D0 brane. However, the remaining supercharges are not broken, as they can no longer be defined in the asymptotic

[1] It is worthwhile to explain why we do not get an analogous partitioning in the case of SU(N) gauge theories. The point is that these bound states in the SU(N) case are interpreted as 11 dimensional gravitons. A state with many gravitons has a relative wave function which is a plane wave and is therefore not normalizable. In the SO(N) and Sp(N) cases, there is no relative wave function since all states live at a point in $\mathbb{R}^9$.

space of the orientifold.[2] There is therefore no multiplet of D0 brane states in this theory; the D0 brane is a supersymmetry singlet. (The fact that the D0 brane in type IIA on $\mathbf{T}^9/\mathbb{Z}_2$ is a supersymmetry singlet was also pointed out in [15].)

We have therefore been led to postulate a chiral fermion living on the orientifold singularity. This fermion is a singlet under the preserved supersymmetries.

The Hilbert space of this single chiral fermionic field can be constructed straightforwardly.

The fermion on the $\mathbf{S}^1$ can either be in the R or NS sector i.e. it can be periodic or antiperiodic, respectively.

In the NS sector, the fermion can be expanded into half-integer modes $b_{n+1/2}$. These act on the vacuum denoted by $|0\rangle$.

For total mode number $k \in \mathbb{Z}/2$, the Hilbert space is the set of states given by $b_{n_1+1/2}b_{n_2+1/2}\cdots b_{n_m+1/2}|0\rangle$, with $k = (n_1+1/2)+(n_2+1/2)+\cdots+(n_m+1/2)$. Here the n_i are distinct by the exclusion principle. One should recall here that the mode number actually counts the number of physical D0 branes. That is, a state with $n+1/2$ mode number corresponds to $2n+1$ half physical branes. k can be either integral or half integral depending on whether m is even or odd. For the m even case we can rewrite the above equation as $N = (2k) = (2n_1+1)+(2n_2+1)+\cdots+(2n_m+1)$. For the m odd case we can rewrite the equation as $N = (2k+1) = (2n_1+1)+(2n_2+1)+\cdots+(2n_m+1)$.

Hence the number of states with N half D0 branes is the number of partitions of N into distinct odd integers. Note that the states are always built from an odd number of half D0 branes.

This is identical to the spectrum found by Kac and Smilga for SO(N) gauge theories.

The R-sector is very similar. The new thing here is that there is a zero mode b_0. Accordingly there are two vacuum states: $|0\rangle$ and $b_0|0\rangle$.

We can build the Hilbert space by acting by any distinct combination of b_n. The Hilbert space at mode number k is then the set of states $b_{n_1}b_{n_2}\cdots b_{n_m}|0\rangle$ and $b_{n_1}b_{n_2}\cdots b_{n_m}b_0|0\rangle$. Here again n_i are distinct and $k = n_1+n_2+\cdots n_m$. which we rewrite in physical D0 brane number as $N = 2k = 2n_1+2n_2+\cdots+2n_m$.

The number of states in each vacuum sector is then the number of partitions of N into distinct even integers. This is identical to the spectrum found by Kac and Smilga for Sp(N) gauge theories.

Furthermore, we have two vacuum states. These presumably correspond to the two types of Sp(N) orientifold planes, the case with no b_0 state corresponds to the familiar O^+ point and the case with the b_0 state corresponds to the more exotic $\widehat{\mathrm{O}0}$ point.

[2]We would like to thank J. Polchinski for a discussion of this point.

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4. More on M theory

We can actually find confirmation of these ideas through a remarkable analysis of Dasgupta and Mukhi [10], who considered compactifications of M theory on $\mathbf{T}^9/\mathbb{Z}_2$. This theory has (0,16) supersymmetry in 1+1 dimensions.

Let us review their analysis. Eleven dimensional supergravity contains 128 bosonic degrees of freedom and 128 fermionic partners. One dimensionally reduces on $\mathbf{T}^9/\mathbb{Z}_2$ and identifies the untwisted sector, which is made of a gravity multiplet and 128 dimensionally reduced scalars accompanied by 128 right moving fermionic partners. These scalars and fermions are arranged in 16 supermultiplets of (0,16) supersymmetry in 1+1 dimensions. This matter content is anomalous. Computation shows that the gravity multiplet has 3 times the anomaly of the 16 matter supermultiplets. In total we need $128 \cdot (1+3) = 512$ left moving fermions to cancel the anomaly. This suggests that each orbifold line has a left moving fermion on it, since there are 2^9 fixed points for the orbifold action (being left moving means that it can be excited without destroying the right moving supersymmetry).

This is in exact agreement with what we find.

We can furthermore (following [11]) compute the ground state energy of this compactification. This is nonzero, because of the Casimir energy of the chiral fermions and chiral bosons.

We need to know for this calculation the Casimir energy of

a) a periodic boson: $-1/24R$

b) a periodic fermion: $1/24R$

c) an antiperiodic fermion: $-1/48R$.

Now the right moving fields are in supermultiplets, and do not contribute to the Casimir energy. Among the left-movers, in the NS case, there are 128 periodic bosons and 512 antiperiodic fermions. The total energy is then $-16/R$. Since there are 512 fixed points, the energy per fixed point is then $-1/32R$. A similar calculation in the R sector yields an energy per fixed point equal to $1/32R$.

This is in precise agreement with the masses of $O0^-$ and $O0^+$ orientifold points. Note that there is a bulk contribution to the mass, $-1/96R$, from the untwisted sector which is common to all kinds of orientifold points, while the twisted fermion contribution differs.

One of the corollaries of our construction is that all the orientifold points must be realized in M theory as a $\mathbb{R}^9/\mathbb{Z}_2 \times \mathbf{S}^1$ space. For suppose that the $\mathbb{Z}_2$ action did not leave the $\mathbf{S}^1$ invariant. There are two options: the $\mathbb{Z}_2$ can act as a shift, or it can act as a reflection. In the first case, the resulting space is smooth, with no fixed points, and there is no natural way to introduce a new field. In the second case, there are two singular points, not a singular line, and we cannot introduce a one-dimensional field. This only leaves the case that the $\mathbb{Z}_2$ does not act on the $\mathbf{S}^1$.

5. Matrix theory

The preceding discussion leads to a Matrix theory realization of M theory on the space $\mathbb{R}^9/\mathbb{Z}_2$. Explicitly, we find that $\mathrm{SO}(N)/\,\mathrm{Sp}(N)$ gauge theories with 16 supercharges describe DLCQ M theory on $\mathbb{R}^9/\mathbb{Z}_2$ with a fermion in the NS/R sector respectively. In this interpretation, the $\mathbf{S}^1$ is a light-cone direction.

A related proposal for Matrix theory on $\mathbf{T}^9/\mathbb{Z}_2$ was put forward in [13]. The gauge group considered there was SO(32). (It is not possible to take arbitrary N in this case, due to charge conservation in $1+1$ dimensions.) The authors were able to show via parton scattering that the charge of the orientifold was $-1/32$, in agreement with our results.

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